Name: ____________________

MA 1118 - Multivariable Calculus
Quiz I - Quarter I - AY 02-03

Instructions: Work all problems. Read the problems carefully. Show appropriate work, as partial credit will be given. No notes or tables permitted.

---

1. (12 points) Determine whether the given sequence converges or diverges. If the sequence diverges, state why. If the sequence converges, find the limit. (**Note** both of these are *seqences*, **not** infinite series!)

a. $a_n \;=\; \dfrac{3+5n^2}{n+2n^2}$

solution:

Observe that, for "large" n,

$$a_n \to \frac{5n^2}{2n^2} = \frac{5}{2} \qquad \Longrightarrow \qquad \lim \quad n \to \infty a_n = \frac{5}{2}$$

i.e. the sequence converges to $\frac{5}{2}$.

b. $a_n \;=\; \dfrac{\ln\left(n^2\right)}{n}$

solution:

Note that for "large" n,

$$a_n \to \frac{\infty}{\infty}$$

which is indeterminate. But observe that $a_n = f(n)$ for $f(x) = \frac{\ln\left(x^2\right)}{x}$, and by L'Hospital's rule:

$$\lim_{x\to\infty} \frac{\ln\left(x^2\right)}{x} = \lim_{x\to\infty} \frac{\left(\frac{1}{x^2}\right)(2x)}{1} = \lim_{x\to\infty} \frac{2}{x} = 0$$

Therefore the sequence converges to 0.

2. (8 points) Determine whether the given series converges or diverges. If the series diverges, state why. If the series converges, find the limit.

$$\sum_{n=0}^{\infty}(-1)^n \frac{3^{n+2}}{4^n}$$

solution:

Observe

$$\sum_{n=0}^{\infty}(-1)^n \frac{3^{n+2}}{4^n} = \sum_{n=0}^{\infty} 3^2(-1)^n \frac{3^n}{4^n} = 3^2 \underbrace{\sum_{n=0}^{\infty}\left(-\frac{3}{4}\right)^n}_{\text{geometric: } r=-3/4}$$

Since $|r| < 1$, we know this geometric series will converge, and furthermore, that the limit value will be

$$\sum_{n=0}^{\infty}(-1)^n \frac{3^{n+2}}{4^n} = 3^2 \sum_{n=0}^{\infty}\left(-\frac{3}{4}\right)^n = 3^2 \frac{1}{1-\left(-\frac{3}{4}\right)} = 9\,\frac{4}{7} = \frac{36}{7}$$